INTERNAL RESONANCE OF A T-SHAPED ELECTROSTATIC LEVITATION ACTUATOR

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ABSTRACT

In this work, we present a demonstration for the internal resonance of an electrostatically actuated T-shaped micromachined beam. The beam has non-conventional configuration with two side electrodes, a fixed bottom electrode, and a hanging beam. The configuration allows the beam to be excited out of plane. The experiments show that the beam oscillates at double the excitation frequency when excited at 90-95 kHz because of the internal resonance between two modes. To prove this, experimental analysis was done on the beam by measuring the mode shapes and identifying unique modal points, which are expected to oscillate in response to one of the modes but has little motion on the other modes.

KEYWORDS

Nonlinear dynamics, internal resonance, MEMS, electrostatic levitation, T-beam.

INTRODUCTION

Micro-electro-mechanical systems (MEMS) are commonly used for sensing and actuation. Capacitive MEMS sensors are usually made using a parallel-plate design. However, this design has limited travel range and suffers from stiction and pull-in instability. The incorporation of side electrodes to the parallel plate allows the moving beam to move out of plane that eliminates the issue of pull-in instability and allows for a significant increase in the beam travel range. Also, the non-trivial electrostatic configuration opens the door to benefit from various nonlinear dynamics in the actuation, including subharmonic resonance [1,2], parametric resonance [1-3], as well as internal resonance [4], to further improve the sensing or actuation responses and signal-to-noise ratios.

Internal resonance often happens when the ratio between system mode frequencies is close to a commensurate ratio, which can usually be realized from non-trivial actuation or nonlinear distribution of mass elements. In such cases, excitation at one mode frequency will trigger the other mode to much higher amplitudes of vibration because the excitation forces as well as the energy transfer are coupled between this pair of modes. Many studies investigated systems with focus on having internal resonance behavior such as Petit et al. where AFM performance was enhanced with higher modes by using a designed 3:1 internal resonance [5]. A more complicated design was investigated by Yu et al. with stepped silicon beam that changes thickness [6]. Changing thickness of a beam was also incorporated with the utilization of a polymer in the fabrication process in [7]. A comprehensive work on internal resonance of an electrostatic actuator with a fixed-fixed arch resonator was presented by Hajjaj et al. [8].

In this work, the design of a levitation actuator with a T-shaped cantilevered beam is shown. The beam is designed with a 1:2 frequency ratio between the higher out-of-plane bending modes. The T-shaped beams were designed according to POLYMUMPS standard, which makes them easy to fabricate with micro-machining technology.

The nonlinearity of the actuation force and the choice of design of the dimensions of the T-shape result in internal resonance behavior with unique properties that are ideal for band-pass filters. These properties include a wide bandwidth with rapid roll-off from the passband to stop-band. Internal resonance can be used to stabilize resonators and broaden their bandwidth for applications in atomic force microscopes. Compared with most recent works on internal resonance by T-shaped microstructure, [9-10], our configuration eliminates any additional rigid masses or magnets and simplifies the design by having uniform thickness using T-shaped beams.

DESIGN

The fundamental design of the T-shaped electrostatic levitator is outlined in reference [1]. This design consists of a primary beam that is 500 μm in length, with a secondary beam attached perpendicular to its tip. Both two beams are designed to be of same 20 μm width for robustness in fabrication and testing. To ensure successful internal resonance from mode shape relations, the length of secondary beam is varied from 50 μm to 300 μm as in Table 1. Finite element analysis in COMSOL is conducted to find the first six mode shapes for each T-shaped beam. Here the “bend”, “tilt”, and “twist” terms represent the specific modes of vibration: out-of-plane bending, in-plane bending, torsional motion along the longitudinal axis of the primary beam, respectively.

An important finding is that when the length of secondary beam’s length does not exceed 200 μm, a consistent 1:2 ratio is observed between two neighboring bending modes, with frequencies around 100 kHz and 200 kHz. However, if the secondary beam is too short, such as 50 μm or 100 μm, actuating bending the mode at around 100 kHz possibly causes the undesired modes in the vicinity where the beam would tilt (for 50 μm case) or twist (for 100 μm case). Secondary beam of 200 μm long is better than 150 μm long because the bending mode at around 100 kHz is farther from the tilting mode and the ratio 1:1.998 between the bending modes in internal resonance is closer to 1:2.

Therefore, a secondary beam of 200 μm long is recommended to avoid the unwanted interactions between bending and twisting or tilting modes. This choice facilitates this investigation of the 1:2 internal resonance phenomenon in bending modes. The complete device dimensions are summarized in Table 2.

Table 1. T-beam natural frequencies (unit: kHz) and mode shapes with varied secondary beam lengths.

Mode	50 μm	100 μm	150 μm	200 μm	250 μm	300 μm
1	6.517 (bend)	5.710 (bend)	5.140 (bend)	4.712 (bend)	4.375 (bend)	4.099 (bend)
2	41.906 (bend)	39.111 (bend)	37.601 (bend)	36.634 (bend)	26.701 (twist)	20.239 (twist)
3	86.724 (tilt)	75.894 (tilt)	57.539 (twist)	37.410 (twist)	35.931 (bend)	35.356 (bend)
4	119.27 (bend)	103.84 (twist)	68.151 (tilt)	62.212 (tilt)	57.423 (tilt)	53.418 (tilt)
5	236.61 (bend)	114.26 (bend)	111.75 (bend)	109.89 (bend)	107.77 (bend)	103.28 (bend)
6	243.46 (twist)	229.74 (bend)	225.76 (bend)	219.54 (bend)	184.91 (bend)	138.37 (bend)

Table 2. T-beam dimensions.

Parameter	Symbol	Value
Primary beam length	L	500 µm
Secondary beam length	L_T	20 µm
Primary beam width	b	20 µm
Secondary beam width	b_T	200 µm
Beam thickness	h	1.5 µm
Initial gap	g	2 µm

RESULTS

Electrostatic levitation actuation

With the mode shapes in the specially designed T-beam actuator, the cubic and quadratic nonlinearities from electrostatic levitation enable the two modes to interact and transfer energy from one to the other causing internal resonance behavior. The beam bends upward when the side (red) electrode is excited (schematic in Figure 1 (a) because of electrostatic levitation. The optical profiler image of Figure 1 (b) shows the actuator curls-up as the side electrode is excited.

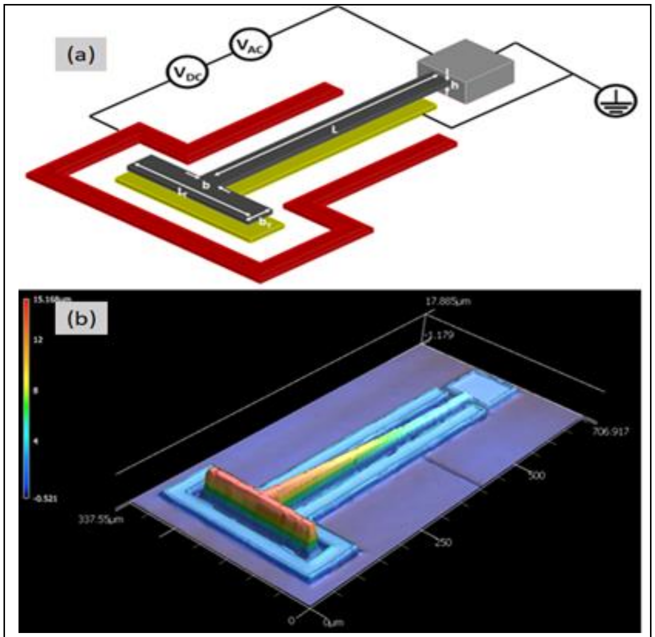

Figure 1: (a) T-beam schematic. (b) Optical profilometer image of the T-shaped beam levitation actuator with actuation of 60V DC on side electrodes.

The experimental setup is shown in Figure 2. A vacuum chamber is used to minimize damping, a DAQ and an amplifier are used to excite the T-beam, and the velocity and mode shapes were measured by a laser doppler vibrometer.

Two bending modes for the T-beam are measured using the vibrometer with chirp excitation and the experimental results showed the frequencies of 92.2 kHz and 183.6 kHz and their ratio is close to 1:2. The simulated, from COMSOL, and measured mode shapes are shown in Figure 3. There is close between the results predicted by the FEA and the experimental measurements.

Frequency response

The device was subjected to excitation with 100 V in DC and 4-10 V AC in the range of 90-95 kHz. Then, the device motion was recorded, and a Fast Fourier Transform was acquired to study the response spectrum. The internal resonance of 1:2 was observed between the fifth and sixth modes at around 92.5 kHz and 185 kHz (Figure 4). We observed typical internal resonance curves in both frequency responses.

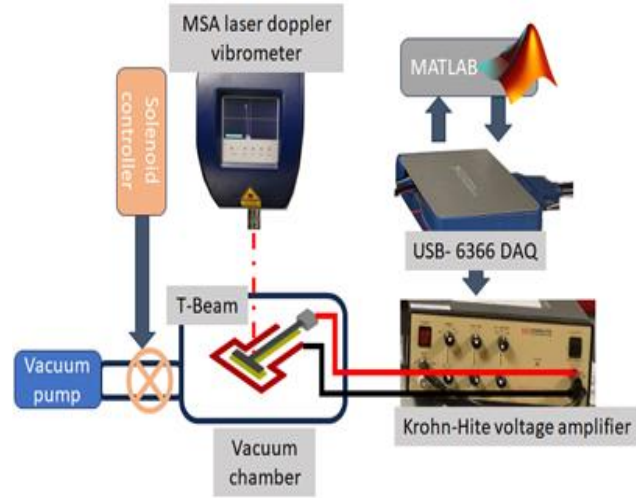

Figure 2: Experimental setup schematic.

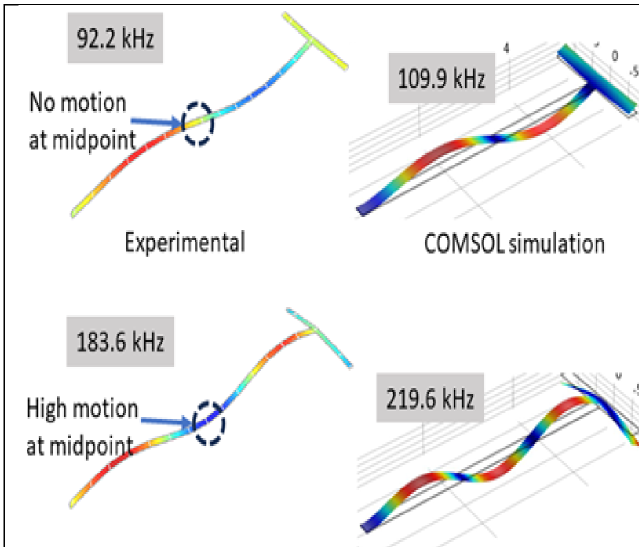

Figure 3: T-shaped beam experimental and simulation mode shapes for the fifth mode and the sixth mode.

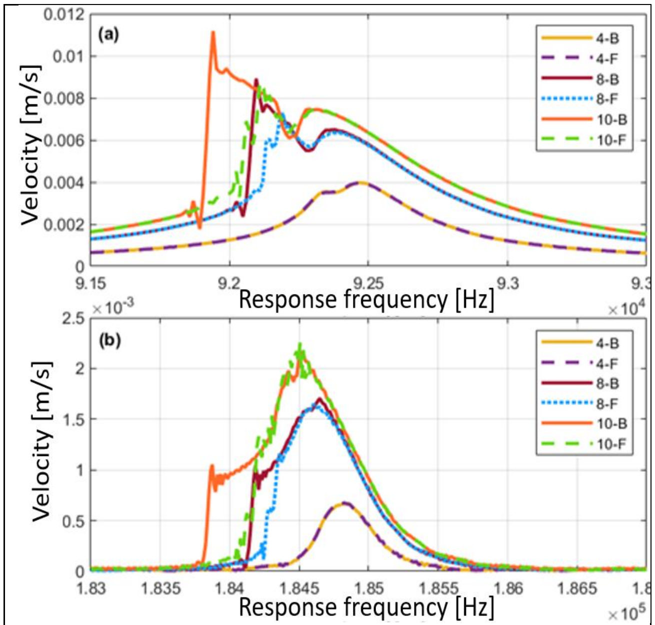

Figure 4: Internal resonance for excitation around 92.5 kHz. (in the legend the numerical value is the AC excitation voltage, B and F annotate backward and forward sweep, respectively). (a) Response frequency in the vicinity of 92.5 kHz, and (b) Response frequency in the vicinity of 185 kHz.

In both cases, the beam reveals only bending motion and the exchange of energy between different bending mode shapes can explain the internal resonance phenomenon. To prove that this is internal resonance, we took the measurement at the midpoint which is highlighted in both modes in Figure 3 to take advantage of the fact that this point motion is negligible in the fifth mode but is significant in the sixth mode. Once the system is excited at the fifth mode, the fast Fourier transform (FFT) and the time history of the response were acquired and are shown in Figure 5. It is noticed that the peak at the sixth mode is higher than the peak at the driven mode, the fifth mode, for the midpoint. The time history shows the midpoint oscillation frequency is 183 kHz (sixth mode frequency). This indicates the transfer of energy from the fifth mode to the sixth mode that originates from the inherent quadratic nonlinearity and the commensurate 1:2 ratio between two modes proving its internal resonance behavior.

CONCLUSION

In this work, we presented experimental results of internal resonance of an electrostatic actuator with T-shaped beams. The beam is designed such that an intrinsic ratio of 1:2 exists between the natural frequencies of two bending modes: the fifth mode and the sixth mode. The experiments have shown that two bending modes exist in 92.2 kHz and 183.6 kHz with ratio of 1:1.998. When excited around the fifth mode with 4-10 V AC and 100 V DC around 92.33 kHz, the beam shows internal resonance behavior with a significant response at 184.51 kHz.

The beam exhibits exchange of energy between the different bending modes, this was proved by measuring the response at midpoint of the beam which does not move in the mode corresponding to 92.2 kHz mode shape but moves significantly in the other mode. The midpoint excitation resulted in a significant peak in the FFT for the higher frequency and the time history of the response shows doubling the frequency of the excitation.

The presented design of the T-beam resonator has advantages over other works because of the simplicity of the design and because it was fabricated with PolyMUMPS standards. The intrinsic internal resonance is solely created by manipulating the dimensions of the T-beam rather than complex structures or including materials that complicate the fabrication process.

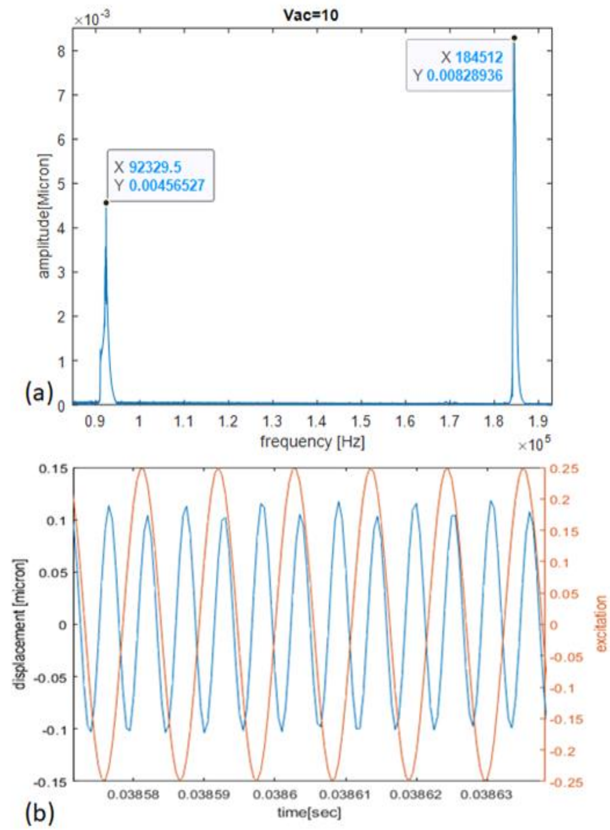

Figure 5: (a) FFT of the response at the midpoint highlighted in figure 3 when exciting at 92.3 kHz, (b) the time history of the midpoint.

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