WHAT IS EFFECTIVE QUALITY FACTOR?

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INTRODUCTION

Many mechanisms have emerged for tuning the effective quality factor (Q_{eff}) of micro- and nano-electromechanical (MEM/NEM) resonators, including parametric amplification [1], thermal-piezoresistive pumping (TPP) [2], external velocity-proportional feedback [3], piezoelectric amplification [4], and optomechanical back-action [5]. These techniques have far-reaching applications for improving the signal-to-noise ratio (SNR) and bandwidth of MEM/NEM sensors. But these methods raise questions about the relationship between the mechanical quality factor, the effective quality factor, and the noise within a resonator.

The quality factor, Q, is an inverse measure of the dissipation of a resonator [6], and determines the mechanical transfer function and the mean squared thermal noise force via the fluctuation-dissipation theorem [7]. The larger the dissipation, the larger the variance in the thermal noise force. The effective quality factor, Q_{eff}, results from a third terminal source or sink of energy, which modifies the resonator transfer function but not the thermomechanical noise force. The ultimate limit to resolution of a resonant sensor is imposed by the thermal noise floor of the underlying MEM/NEM resonator, which can be reduced by increasing Q, but not Q_{eff} [8]. It is therefore important to distinguish between changes in Q and Q_{eff} in a MEM/NEM resonator.

The most common technique for measuring Q of a MEM/NEM resonator is the bandwidth (or 3-dB) method, given by:

$$Q = \frac{\omega_n}{\Delta\omega}, \tag{1}$$

where ω_n is the resonant frequency and $\Delta\omega$ is the linewidth of the resonance. The bandwidth method uses the resonant frequency and the width of the driven response peak at half-max to estimate Q. The second technique is the Lorentzian fit, given by:

$$A(\omega) = \sqrt{G\frac{4k_BT\left(\frac{\omega_n}{mQ}\right)}{({\omega_n}^2-\omega^2)^2+\left(\frac{\omega\omega_n}{Q}\right)^2}+N_a^2}, \tag{2}$$

where $A(\omega)$ is the amplitude spectral density (ASD) of the voltage at the amplifier output for a resonator driven only by its intrinsic thermal noise, G is the squared scale factor between resonator displacement and amplifier output, m is the lumped mass of the mode, k_B is Boltzmann's constant, T is the resonator bulk temperature, and N_a is the amplifier noise floor. In Eq. 2, the denominator of the first term corresponds to the mechanical transfer function while the numerator corresponds to the thermomechanical noise force. To extract Q, the Lorentzian fit method fits Eq. 2 to the displacement noise of the resonator at resonance, which requires a very sensitive displacement readout. Often the displacement transduction noise greatly exceeds the resonator thermal noise, so an external driving force must be used, which invalidates Eq. 2.

Q_{eff} tuning modifies the denominator of the first term in Eq. 2, but not the numerator, which results in an ASD given by:

$$A_{eff}(\omega) = \sqrt{G\frac{4k_BT\left(\frac{\omega_n}{mQ}\right)}{({\omega_n}^2-\omega^2)^2+\left(\frac{\omega\omega_n}{Q_{eff}}\right)^2}+N_a^2}\,. \tag{3}$$

Fig. 1 compares the resonator thermomechanical displacement noise at resonance by plotting Eqs. 2 and 3. Modifying Q or Q_{eff} modifies the linewidth of the mechanical transfer function in the same manner, while only a change in Q changes the thermal noise floor.

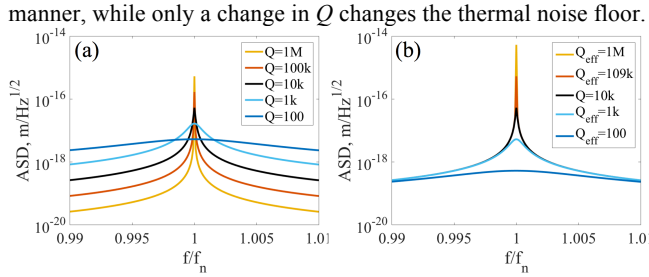

Figure 1: Displacement noise amplitude spectral density (ASD) near resonance for changing (a) Q, and (b) Q_{eff}. This is obtained by plotting Eqs. 2 and 3, respectively, for zero amplifier noise ($N_a = 0$) and unity amplifier gain ($G = 1$).

EXPERIMENTS

We experimentally illustrate the difference between Q and Q_{eff} using a silicon resonator subjected to parametric amplification and TPP. We fabricated the device shown in Fig. 2 within a wafer-scale encapsulation process [9]. Our setup allows us to simultaneously bias the device for capacitive actuation and sensing while flowing a direct current to induce TPP. We can alternately parametrically amplify the resonator motion by applying a voltage at $2\omega_n$ to the drive electrode to modulate the electrostatic spring constant.

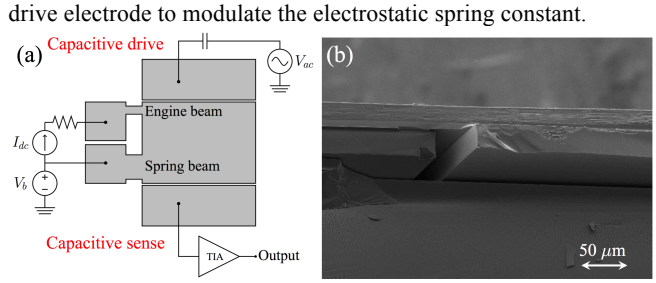

Figure 2: (a) Measurement setup. We bias one anchor of the resonator at V_b and flow a current I_{dc} through the device. (b) Cross-section of our device after encapsulation.

Fig. 3 demonstrates parametric amplification and TPP using an open-loop sweep. For increasing parametric pump or increasing direct current, the resonator amplitude increases and the linewidth decreases, which corresponds to an increase in Q_{eff} using the bandwidth method. Because we use an external driving force, we can only measure the resonator mechanical transfer function, so changes to Q or Q_{eff} are indistinguishable.

To show that TPP and parametric amplification tunes Q_{eff}, not Q, we must study the thermomechanical motion of the resonator directly. We remove the external drive and use a custom low-noise transimpedance amplifier (TIA) and a large bias voltage to capacitively detect the displacement fluctuations driven by the thermal noise force. Fig. 4 shows the ASD of the amplifier output at resonance, measured using a scalar spectrum analyzer, as we apply a progressively larger parametric pump and zero direct current. As the pump is increased, the thermal noise at resonance is increased while the noise away from resonance does not change: the ASD in

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Fig. 4 qualitatively matches the ASD in Fig. 1 (b), not Fig. 1 (a). This clearly shows that parametric amplification only modifies the mechanical transfer function, not the thermal noise force, so parametric amplification tunes Q_{eff}, not Q. In Fig. 5, we repeat this measurement by switching off the parametric pump and increasing the direct current through the resonator. We again see that Fig. 5 matches Fig. 1 (b), not Fig. 1 (a), so TPP tunes Q_{eff}, not Q. Eq. 3 fits the resonator noise excellently in both Fig. 4 and Fig. 5 with a constant Q and more than a ten-fold increase in Q_{eff}.

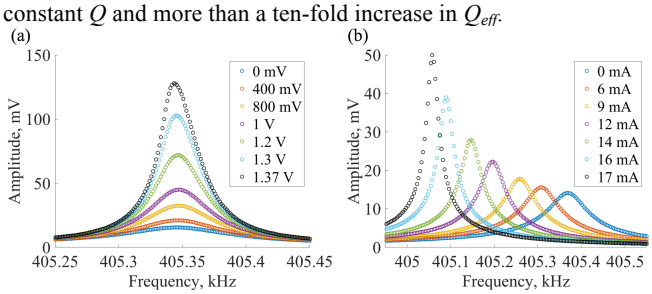

Figure 3: Open-loop amplitude sweep of our device subjected to (a) degenerate parametric amplification, and (b) thermal-piezoresistive pumping (TPP). The frequency downshifts for TPP because of the heating that accompanies a direct current through the device.

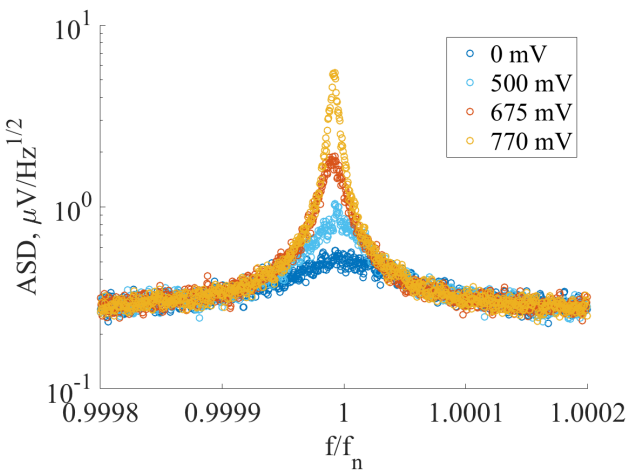

Figure 4: Amplitude spectral density (ASD) of thermomechanical displacement fluctuations of our resonator for increasing parametric pumping voltage, which induces degenerate parametric amplification.

Parametric amplification differs from other Q_{eff} tuning mechanisms because it is quadrature-specific; the thermal motion in-phase with the parametric pump is amplified while the motion anti-phase to the pump is suppressed [1]. For TPP and other linear feedback mechanisms, the noise motion is amplified equally for all phases [10]. Because we measure the displacement noise magnitude in Figs. 4 and 5 with averaging, Fig. 4 corresponds to an ensemble average of parametric Q_{eff} enhancement in-quadrature and parametric Q_{eff} suppression in the anti-quadrature, which still only modifies the resonator transfer function, not the thermal noise force.

CONCLUSION

Parametric amplification and TPP modifies Q_{eff}, not Q. Readers should be skeptical of claims of mechanical Q tuning in a MEM/NEM resonator with a third terminal. Credible claims should be accompanied by a measurement of the thermal displacement noise that matches Fig. 1 (a), not Fig. 1 (b).

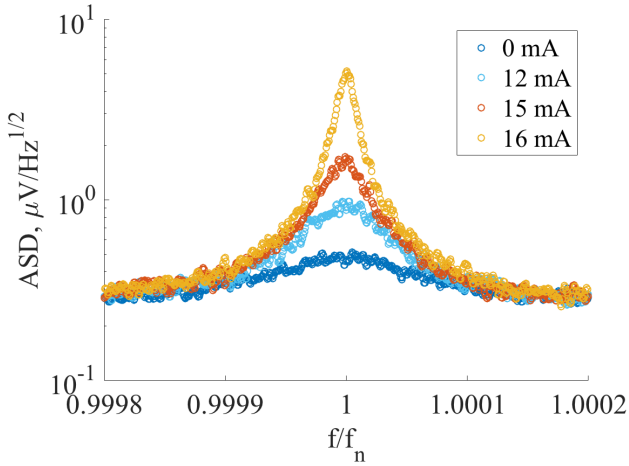

Figure 5: Amplitude spectral density (ASD) of thermomechanical displacement fluctuations of our resonator for increasing direct current, which induces thermal-piezoresistive pumping.

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