PHASE NOISE REDUCTION IN AN OSCILLATOR THROUGH COUPLING TO AN INTERNAL RESONANCE

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ABSTRACT

In this paper, we describe an oscillator, operating at room temperature that can be operated in a condition of internal resonance, where a driven, in-plane mode of the MEMS frequency selective resonator interacts with a torsional mode, resulting in 70 dB decrease in phase noise at a 1 Hz offset as compared to the oscillator operating in driven mode alone. The resonator element is a clamped-clamped beam where the primary mode of oscillation is an in-plane flexural mode, which can be driven at a frequency where vibrational energy is coupled to a higher frequency torsional mode. The coupling to the torsional mode stabilizes the vibrational frequency of the primary mode, resulting in a measured phase noise of -90 dBc at 1 Hz offset and an Allan deviation of 4 x 10^{-9}. These oscillators show similar behavior to quartz crystals and could be explored for use in timing applications where, currently, single mode resonator micro- and nano-mechanical oscillators are being used in applications such as clocks and frequency standards. We present a theoretical model that qualitatively explains the behavior and demonstrates that phase noise can be greatly reduced at the internal resonance condition.

INTRODUCTION

Oscillators are a key component in practically all electronic devices, serving as clocks for logic circuits, frequency standards for heterodyning communication signals, timing applications, and as sensors [1-3]. Currently, most commercial oscillators are fabricated from quartz crystals, which are macro-scale, cannot be integrated with the CMOS circuitry, and consume a relatively large amount of power. Microelectromechanical (MEMS) oscillators are being produced as alternatives to quartz oscillators primarily because they offer a path to create a monolithic structure with all circuit elements fabricated on a single silicon chip using fabrication techniques that are compatible with CMOS processing [4]. Commercial MEMS oscillators are starting to show up in uses such as gyroscopes and timing applications. Most MEMS oscillators operate in the linear regime of the resonator element; however, as the dimensions of the MEMS are reduced to keep in step with the CMOS circuitry, the linear operating range is reduced [5]. MEMS oscillators have typically not operated in the non-linear regime because non-linear oscillators experience increased frequency noise due to amplitude-frequency noise conversion through the amplitude dependence of frequency, generically modeled by a cubic (Duffing) restoring force [6]. In spite of these detrimental effects, it has been recently demonstrated that non-linear resonator dynamics can be used to reduce phase noise in oscillators [7-9]. We have previously showed that a non-linear oscillator operating at an internal resonance condition shows improvement in frequency stability by a factor of over 300 [9]. In this paper, we present a detailed study of the phase noise reduction achievable in the current MEMS devices and we introduce a model to describe the mechanism responsible for this reduction in phase noise.

DESIGN, FABRICATION AND OPERATION

The resonators are fabricated using the SOIMUMPS process from MEMSCAP with a 10 μm thick silicon layer. The resonator consists of 500 μm long, parallel, doubly clamped beams attached at their center to ensure that the three beams move together as a single element (Fig. 1).

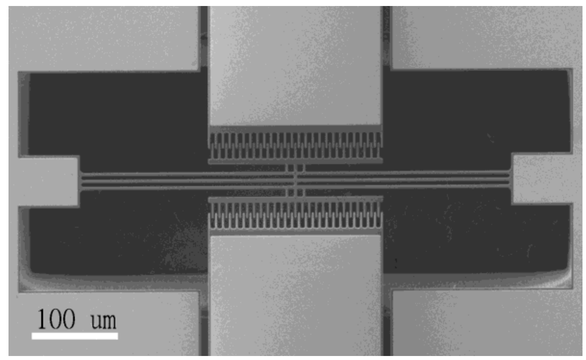

Figure 1: Scanning electron micrograph of the oscillator showing the comb drives for actuation and signal transduction.

Attached to the center of the beam are two comb drives. One comb drive is used for driving the resonator while the other comb drive is used to transduce its response. The total capacitance of each comb drive is calculated to be 50 fF. The primary mode of oscillation is an in-plane flexural mode with a linear response at a frequency of approximately 65 kHz. The second and third order modes are a flexural out-of-plane and torsional mode respectively, both with a modal frequency of approximately 200 kHz [9]. The electronic operation of the device has been described previously [9] and will be summarized briefly. An AC voltage is applied to one of the comb drives, which causes the beam to vibrate at the frequency of the AC voltage. The motion of the beam causes a changing capacitance in the other comb drive, which generates an AC current at the output of the comb drive. The current is passed through a current preamplifier, where it is filtered by a band pass filter between 10 kHz and 1 MHz. The output of the preamplifier is sent through a transimpedance amplifier to convert the current into a voltage. The voltage output is phase shifted and used as the reference in a phase feedback oscillator [3], where the drive voltage amplitude is held constant, independent of the oscillator amplitude. The output from the transimpedance amplifier is also used to measure the phase noise and the Allan deviation of the oscillator through a Zurich UHFLI lock-in amplifier. The phase noise is measured by the power spectral density (PSD) of the in-phase and quadrature components of the voltage signal at a specified frequency. Experimentally, the reference frequency of the lock-in amplifier is set so that the phase of the measured signal does not deviate for a short period of time. Then the PSD is acquired over a period of time, the inverse of which, defines the frequency resolution of the spectrum. In a second experiment, the Allan deviation is calculated using a running average of the frequency data recorded as a function of time.

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From an experimental perspective, to achieve oscillation, the oscillator needs to be entrained by sweeping the drive voltage from a start frequency to the desired frequency where the drive can be switched to the phase feedback circuit. Without sweeping the applied voltage, the oscillator will not be entrained. Once in oscillation, the drive amplitude and phase are used to tune the frequency of the oscillator.

In our system, an internal resonance occurs when the frequency of the primary mode equals 1/3 of the frequency of the torsional mode (1:3 mode coupling). In this situation the coupling between modes acts as a *mechanical negative feedback* responsible for stabilizing the frequency of the resonator [9].

RESULTS

Over a range of operating parameters the system is bi-stable, with one operating condition dominated by the primary mode and the other having two modes active via the internal resonance. To determine the stabilization effect the internal resonance condition has on the oscillator, the phase noise of the oscillator outside and inside the internal resonance condition is measured. The oscillator is set to operate with a 100 mV AC signal and the phase noise of the oscillator is measured using the lock-in amplifier. The phase noise performance of the self-sustained oscillator in and out of the internal resonance condition can be seen in Fig. 2a. When the oscillator is operating outside internal resonance, the phase noise at 1 Hz offset is approximately -20 dBc (using an 8^{th} order, low pass filter and 100 Hz bandwidth for the lock-in). The phase noise shows $1/f^4$ behavior, indicative of frequency random walk. This result is the typical response from a non-linear oscillator where the spectrum is dominated by amplitude-frequency noise resulting from the cubic term in the restoring force.

To measure the effect of the internal resonance condition, the frequency of the oscillator is changed so that the internal resonance condition is met, for this oscillator, at 66520 Hz. The phase noise of the oscillator working at internal resonance is shown in Figure 2a. The phase noise at a 1 Hz offset is approximately -90 dBc (using an 8^{th} order, low pass filter and 10 Hz bandwidth for the lock-in), a reduction of 70 dB over the noise measured outside of internal resonance. The data shows slopes of $1/f^2$ and $1/f^6$. The slope of $1/f^2$ is from the Leeson effect suggesting that the oscillator noise is limited by the noise of one of the amplifiers in the feedback circuitry [10], which indicates that by improving the experimental setup the noise can be reduced even further. From measurements of frequency data as a function of time, the Allan variance was calculated inside and outside the internal resonance condition (Fig. 2b). Frequency stability of 4×10^{-9} at a characteristic time of 10 s can be easily achieved with the device. For timescales above this, the data is limited by the random walk of the oscillator.

THEORY

A fundamental understanding of the oscillator phase noise is the key to identifying the ultimate frequency stabilization achievable by coupling resonator modes through an internal resonance. We have developed a non-linear model to describe the mechanism that reduces the phase noise of the primary mode of the oscillator when it couples to the higher frequency mode. The model is a standard closed-loop oscillator with a frequency selective element consisting of two electro-mechanical vibration modes with a 1:3 frequency ratio between the driven and secondary modes, and these modes are coupled through nonlinear electromechanical effects that arise from finite mechanical deformations.

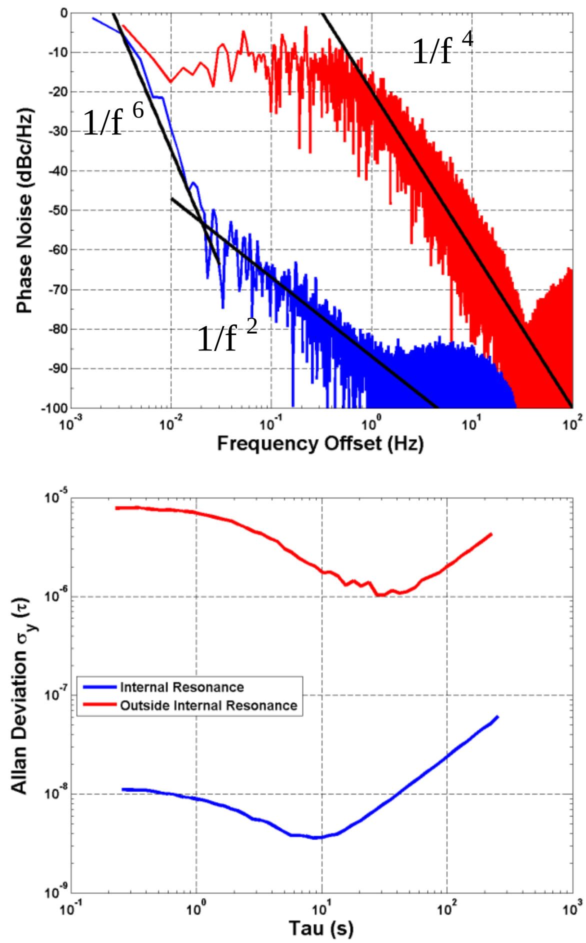

Figure 2: a) Phase noise data from the oscillator operating both outside and inside the internal resonance condition. The change in phase noise is expected from the model of the system. b) A plot of the Allan Deviation away from internal resonance and at internal resonance for different characteristic times, τ. At internal resonance, the minimum Allan Deviation, $4x10^{-9}$, occurs at a characteristic time of 10s. For greater times, the random walk of the oscillator dominates.

The resulting model can be understood as a specific case of a generic two-mode system with non-linear coupling that promotes energy transfer between modes [5]. Additive and multiplicative noise sources are included for both modes, and the feedback loop is modeled using a phase shifter and an amplifier operating in its saturated regime. Using standard perturbation methods, one can derive expressions for the dynamics of the slowly varying amplitudes and phases of the two modes in the closed loop system. Solution of the steady-state operating conditions shows a parameter range where the response is bi-stable, with one stable operating condition dominated by the primary mode and the other involving both internally resonant modes. By linearizing the model about these steady states we can compute and compare the spectrum of the oscillator phase fluctuations and investigate the manner in which system parameters and various noise sources contribute to the phase noise of the oscillator.

For numerical investigations of the model, we employ generic parameters that demonstrate the behavior of interest, since the full set of device system parameters is yet to be determined. White noise with a constant intensity is assumed for all sources, and the intensity for the primary mode is taken to be two orders of magnitude larger than that for the second mode, due to effects of the amplifier, which acts directly on the primary mode. Figure 3a shows the oscillator frequency ω_0, normalized by 1/3 of the second mode frequency, versus the amplifier saturation amplitude F (proportional to the driving voltage). As F is increased, ω_0 initially increases until it becomes close to 1/3 of the secondary mode frequency ω_2, initiating the internal resonance. Through the internal resonance region, the oscillator frequency remains entrained close to this value (red curve), while the frequency of an oscillator operating without internal resonance would continue its upward trend, in the bi-stable region and beyond (black curve). Note that hysteresis is observed when F is swept up and down.

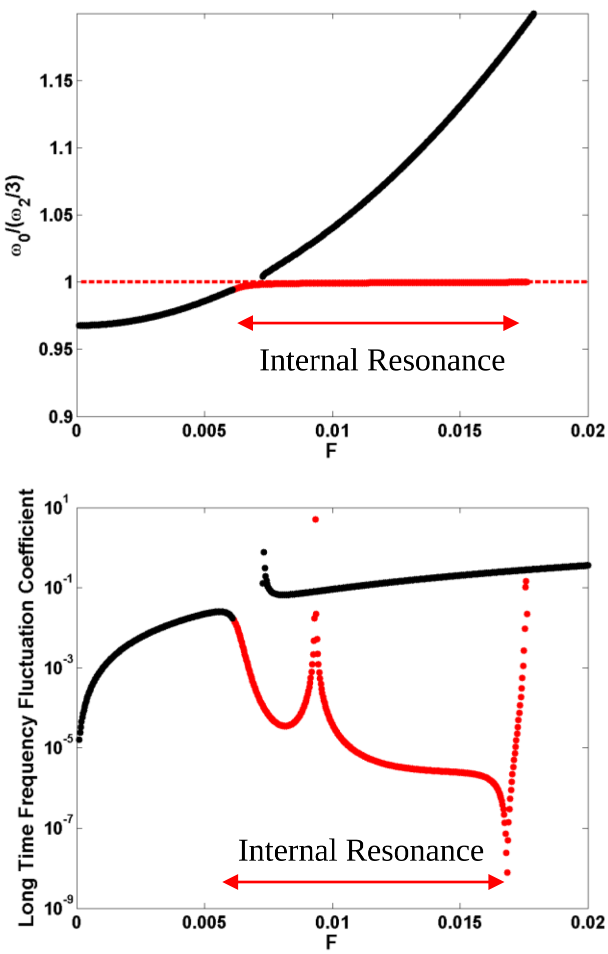

Figure 3: Results obtained from the two-mode oscillator model. a) Normalized oscillator frequency vs. the amplifier saturation amplitude F. As F increases the oscillator frequency increases monotonically along the single mode response (black curve), and remains very close to 1/3 that of the secondary mode when operating with internal resonance (red curve). b) Magnitude of the long term phase diffusion coefficient vs. F. The red curve corresponds to the internally resonant operating condition, while the black curve is from the single mode operating condition. The observed reduction in phase noise due to the internal resonance is consistent with the experimental results reported in [9].

Figure 3b shows the computed magnitude of the long term phase diffusion coefficient versus F over the relevant parameter range. In the range of bi-stability, aside from a narrow peak, the phase noise is reduced by several orders of magnitude when operating with internal resonance, as compared to that without internal resonance. These results are consistent with experimental results presented in [9].

The analysis indicates that the oscillator phase noise is not limited by the phase noise of the primary mode, as is the case with single mode oscillators. In order to utilize this predictive analysis for a given device, one must have values for the system parameters and information about the spectrum of various noise sources, work that is underway for the present device.

CONCLUSION

We present data showing that by operating an oscillator at a frequency where the primary mode of the resonator couples to a higher frequency mode (internal resonance condition), the phase noise of the oscillator can be reduced by orders of magnitude. Characterization of the oscillator shows behavior consistent with a nonlinear two-mode resonator as the frequency selective element. A coupled-mode oscillator model using Duffing-like behavior qualitatively predicts this behavior. We envision this approach being expanded to other oscillator types and modes to provide a means to stabilize the frequency response of oscillators.

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