DESIGN OF ELECTROSTATICALLY ACTUATED MEMS UNDER UNCERTAINTIES

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ABSTRACT

This work presents a stochastic framework based on stochastic collocation approach to quantify the effect of uncertain parameters on the performance of electrostatic MEMS devices. We consider an example of a MEMS switch to demonstrate that the proposed methodology can be used to design effective and reliable MEMS devices. The approach is straightforward to implement and can be orders of magnitude faster than the Monte Carlo method.

INTRODUCTION

State-of-the-art design methodologies for MEMS are based on deterministic approaches [1-2] where the geometrical and physical properties of the device are assumed to be known precisely. However, low cost manufacturing processes often result in significant uncertainties in MEMS. For example, in the case of the transverse comb drive shown in Figure 1, depending on the manufacturing process used, there is always some uncertainty associated with the geometrical features such as the thickness of the movable fingers or fixed teeth, overlap length between the two set of teeth or material properties such as the Young's modulus etc. The inability to account for these uncertainties during computational design can significantly affect the reliability of MEMS. To this end, we seek to develop a stochastic framework with a twofold objective -- firstly, to quantify the effect of variation in various input parameters on the output parameters such as deflection, pull-in behavior etc., relevant to the design of electrostatic MEMS devices. Secondly, to employ this uncertainty quantification data to identify the parameters which are critical to the device performance.

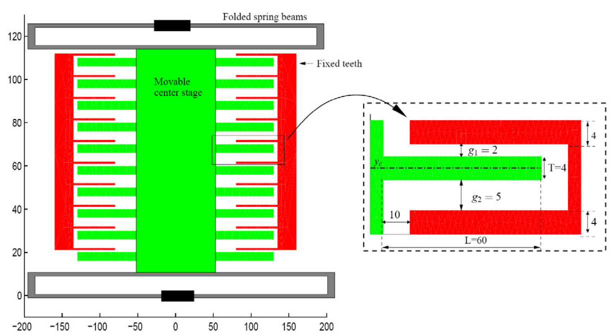

Figure 1: Transverse comb drive.

Typically in MEMS, uncertainties are taken into account through safety factors, which may lead to over conservative designs. Uncertain parameters can also be handled by using sampling based methods such as Monte Carlo (MC) method. For example, the effect of uncertainty in various geometrical parameters on the design of a comb drive has been considered in [3]. The variability in the performance of a ceramic actuator resulting from the variation in the shape of the actuator and the air gap in the condenser has been studied in [4]. Although MC based methods are straightforward to apply and generate the required statistics easily, they can be prohibitively expensive, as their accuracy depends on the sample size. In [5] we presented a framework for stochastic modeling of MEMS based on a spectral discretization technique – *generalized polynomial chaos (GPC)*, which provides fast convergence. However, the GPC based approach leads to a set of coupled deterministic equations that need to be solved, and hence the implementation may be non-trivial (see [5] for details).

In this work, we propose an efficient framework based on the stochastic collocation approach [6-8] to handle uncertain parameters during the design of electrostatic MEMS. The novel features of this approach are – firstly, since it involves sampling at a pre-determined set of points, it is straightforward to implement. Secondly, for the number of uncertain parameters one usually needs to consider for the analysis of MEMS devices, this approach is orders of magnitude faster than the Monte Carlo method to obtain the same level of accuracy. The proposed methodology has been applied to design a MEMS switch under uncertainties.

The paper is organized as follows: first, we present the deterministic formulation for the coupled electromechanical problem. The stochastic formulation and the solution methodology are detailed in next section. In following section we then present a numerical example, and finally conclude the discussion.

DETERMINISTIC FORMULATION

Figure 2 shows a typical MEM device – a deformable cantilever beam over a fixed ground plane. When a potential difference is applied between the beam and the ground plane, it induces electrostatic pressure acting normal to the surface of the beam, which causes it to deflect.

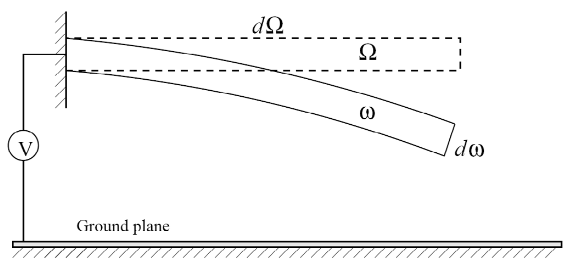

Figure 2: MEM beam under deformation.

Physical level analysis of such a device requires a self-consistent solution of the coupled mechanical and electrostatic equations. A framework for the deterministic analysis is presented in [1]. In the absence of any free charges, the electrostatic potential can be obtained by solving the Laplace equation, given as,

$$\nabla^2\phi = 0 \ \text{ in } \varpi , \tag{1}$$

where ϕ is the potential field in the dielectric medium ϖ, surrounding the conductors. A boundary integral formulation (see [1] for details) of Eq. (1) is used to compute the surface charge density σ on the conductors. The electrostatic pressure P_e acting on the microstructures can be computed from σ as $P_e = \sigma^2 / 2\varepsilon$, where ε is the dielectric constant of the medium.

The mechanical deformation of the microstructure due to the electrostatic pressure is obtained by performing 2-D large deformation elastic analysis. The governing equation for the deformation $\mathbf{u}$, in the absence of any body force, is given as,

$$\nabla \cdot (\mathbf{FS}) = 0 \text{ in } \Omega, \qquad (2)$$

where Ω represents the undeformed configuration of the microstructure, $\mathbf{F} = \mathbf{I} + \nabla \mathbf{u}$ is the deformation gradient and $\mathbf{S}$ is the second Piola-Kirchoff stress tensor. Appropriate displacement and surface traction boundary conditions (from electrostatic analysis) are applied. Eqs. (1) and (2) are solved in a self-consistent manner using relaxation or Newton scheme, as described in [1]. We can represent the coupled electromechanical system as,

$$L(\mathbf{u}, \sigma; \mathbf{x}) = 0 \quad \mathbf{x} \in \Omega. \qquad (3)$$

Such a system can be solved easily using Finite Element method (FEM) and Boundary Element method (BEM). In the next Section, we present the stochastic formulation which employs repetitive usage of such a deterministic solver, in order to generate the desired statistics of various output parameters, relevant to the design of electrostatic MEMS.

STOCHASTIC FORMULATION

The stochastic formulation employs stochastic quantities to model uncertainties – uncertain random parameters are modeled as random variables and uncertain spatial functions are represented as random fields. The basic idea is to treat uncertainty as a separate dimension in addition to space and time (for dynamical problems). The random probability space is characterized using n independent random variables $\boldsymbol{\xi} = [\xi_i]_{i=1}^n$, using which all uncertain parameters are represented as $(n+d)$-dimensional functions, where d refers to the dimension of the physical space. The stochastic formulation for the coupled electromechanical system can be written as: we seek uncertain displacement $\mathbf{u}(\mathbf{x}, \boldsymbol{\xi})$ and surface charge density $\sigma(\mathbf{x}, \boldsymbol{\xi})$, such that

$$L(\mathbf{u}, \sigma; \mathbf{x}, \boldsymbol{\xi}) = 0 \quad (\mathbf{x}, \boldsymbol{\xi}) \in \Omega \times \Gamma, \qquad (4)$$

where Γ represents the random domain.

Monte Carlo method

Monte Carlo simulation has been traditionally used for systems with random input parameters. It involves generating various realizations of the input parameters according to the underlying probability distribution, and repeatedly employing the deterministic solver for each realization. Eq. (4) can be easily solved using MC method as follows:

1. For the given number of samples K, generate independent and identically distributed (iid) random variables $\{\boldsymbol{\xi}^j\} = [\xi_i^j]_{i=1}^n, j = 1, \ldots, K$, according to the underlying distribution.
2. For each of the realizations, solve the deterministic problem $L(\mathbf{u}^j, \sigma^j; \mathbf{x}, \boldsymbol{\xi}^j) = 0$, and obtain the solution $(\mathbf{u}^j, \sigma^j)$, for $j = 1, \ldots, K$.
3. Compute the required statistics such as mean μ and variance ν, for e.g.,

$$\mu(\mathbf{u}) = \frac{1}{K} \sum_{j=1}^{K} \mathbf{u}^j, \quad \nu(\mathbf{u}) = \frac{1}{K} \sum_{j=1}^{K} (\mathbf{u}^j - \mu(\mathbf{u}))^2. \qquad (5)$$

The amount of work required for a MC simulation to converge to achieve the required accuracy τ is given as $\tau(K) = O(1/\sqrt{K})$, which represents very slow convergence.

Stochastic collocation method

The basic idea of the stochastic collocation approach is to approximate the stochastic dependent variables by a polynomial interpolation function in the multi-dimensional random space. The interpolation is constructed by sampling the dependent variables at a pre-determined set of points $\Theta_K = \{\boldsymbol{\xi}^j\}_{j=1}^K$. The set of points Θ_K are chosen as the sparse grid points generated using the Smolyak algorithm (see [6] for details), unlike the MC approach where the sampling points are chosen in a statistical manner. Using the polynomial approximation, the interpolated displacement and surface charge density can be written as:

$$\hat{\mathbf{u}} = \sum_{j=1}^{K} \mathbf{u}(\mathbf{x}, \boldsymbol{\xi}^j) \psi_j(\boldsymbol{\xi}), \quad \hat{\sigma} = \sum_{j=1}^{K} \sigma(\mathbf{x}, \boldsymbol{\xi}^j) \psi_j(\boldsymbol{\xi}), \qquad (6)$$

where $\{\psi_j\}$ are the Lagrange interpolation functions such that $\psi_j(\boldsymbol{\xi}^i) = \delta_{ij}$. Using the interpolated form in the stochastic system given by Eq. (4), we obtain K decoupled deterministic systems (see [7-8] for details),

$$L(\mathbf{u}(\mathbf{x}, \boldsymbol{\xi}^j), \sigma(\mathbf{x}, \boldsymbol{\xi}^j); \mathbf{x}, \boldsymbol{\xi}) = 0, \quad j = 1, \ldots, K.. \qquad (7)$$

The statistics of the random solution, such as mean, can be computed as:

$$\mu(\mathbf{u}) = \sum_{j=1}^{K} \mathbf{u}(\mathbf{x}, \boldsymbol{\xi}^j) w_j, \qquad w_j = \int_{\Gamma} \psi_j(\boldsymbol{\xi}) \rho(\boldsymbol{\xi}) \mathbf{d}\boldsymbol{\xi}, \qquad (8)$$

where $\rho(\boldsymbol{\xi})$ represents the joint probability density function of the random variables $\boldsymbol{\xi}$, and $\{w_j\}$ are the weights that can be pre-computed and stored. The complexity of the sparse grid collocation approach is given as,

$$\tau(K) = O(K^{-r} (\log K)^{(r+1)(d-1)}), \qquad (9)$$

for solutions with bounded mixed derivatives up to order r. Thus, for sufficiently smooth functions, this approach would be orders of magnitude faster than the MC method. In addition to the faster convergence rate, the fact that the procedure only requires solving the deterministic problem at a set of sample points makes the implementation straightforward. The solution methodology has been detailed in Figure 3.

NUMERICAL EXAMPLE

We consider a MEMS switch, modeled by a silicon cantilever beam, which is 80 μm long, 1 μm thick and 10 μm wide, located over a ground plane. As the applied potential difference V between the beam and the ground plane is increased, the beam increasingly deflects towards the ground plane. At a voltage V_p (known as the pull-in voltage), when the electrostatic force can no longer be balanced by the restoring elastic force, the beam finally collapses onto the ground plane, which signifies the *ON* state of the switch. In order to design reliable switches, it is required that we consider the variations in material properties and geometrical parameters, and quantify their effect on device performance parameters such as deflection and pull-in voltage etc.

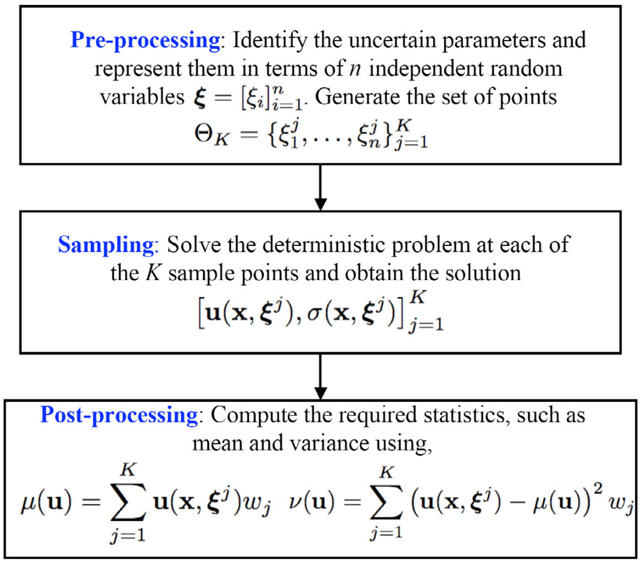

Figure 3: Solution methodology for stochastic collocation method.

Specifically, we consider the effect of uncertain Young's modulus E and the initial gap g between the beam and the ground plane, which are given as:

$$E = E_0(1+\nu_E\xi_1);\; g = g_0(1+\nu_g\xi_2), \tag{10}$$

where $E_0 = 169$ GPa and $g_0 = 1\,\mu m$ are the mean values, $\nu_E = \nu_g = 0.1$, which represents a *10%* variation in both parameters, and $\xi_{1,2}$ are independent uniformly distributed random variables on [-1,1].

Pull-in behavior

The pull-in voltage V_p and the vertical tip deflection at $V = V_p^-$ (deflection just before pull-in), computed by solving the deterministic coupled problem, for various values of E and g are tabulated in Table 1. As can be seen, for the given variation in the parameters, the pull-in voltage varies between 7.3 V and 11.4 V.

Table 1:Pull-in voltage and tip deflection for the cantilever beam.

Sample	E / E_0	g / g_0	$V_p(V)$	Vertical tip displacement (μm)
S0	1.0	1.0	9.2	-0.3431
S1	0.9	0.9	7.3	-0.3007
S2	1.1	1.1	11.4	-0.4217

In Figure 4 we plot the mean vertical tip displacement with various applied potential difference, as obtained from the stochastic collocation approach. We also plot the pull-in curves obtained by solving the deterministic problem for two samples given by S1 and S2. We note that for the applied potential difference higher than the pull-in voltage, the tip displacement is equal to initial gap *g*. Clearly, the realizations S1 and S2 mark the two boundaries for the pull-in curve and are indicative of the worst case behavior of the switch. Thus for all possible values *(E,g)*, the vertical tip deflection of the beam would be between the values indicated by these two curves. In Figure 5 we plot the mean and the error bars, based on standard deviation, for the vertical tip displacement. One could use the information regarding the worst-case behavior to design reliable switches, but such designs may be over conservative. However, the information obtained in terms of the error bars quantifies the variability of the performance of the switch, and may be used to design reliable and efficient switches.

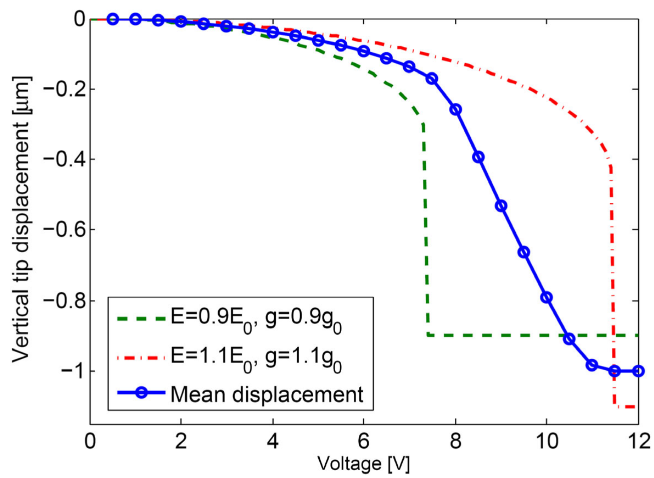

Figure 4: Worst case and mean pull-in behavior for the MEMS switch.

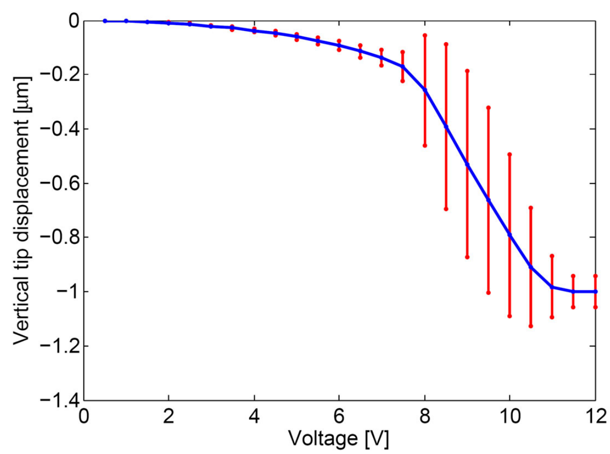

Figure 5: Mean vertical tip displacement with error bars (based on standard deviation).

Design under uncertainties

In addition to quantifying the effect of uncertain parameters on the device performance, the methodology based on GPC (described in [5]) and the stochastic collocation approach can also be used to identify critical design parameters, which one may need to control carefully, in order to improve reliability. In Figure 6 we plot the mean sensitivity of the tip deflection to the variations in Young's modulus $s_E = \frac{1}{\nu_E}\frac{\partial v_{tip}}{\partial \xi_1}$ and gap $s_g = \frac{1}{\nu_g}\frac{\partial v_{tip}}{\partial \xi_2}$ obtained using GPC approach, and conclude that the variations in gap are more critical to the performance of the switch as compared to the Young's modulus.

In Figure 7 we plot the probability of pull-in with various applied voltages obtained using the stochastic collocation approach. As the applied voltage increases from 7.3 V to 11.4 V,

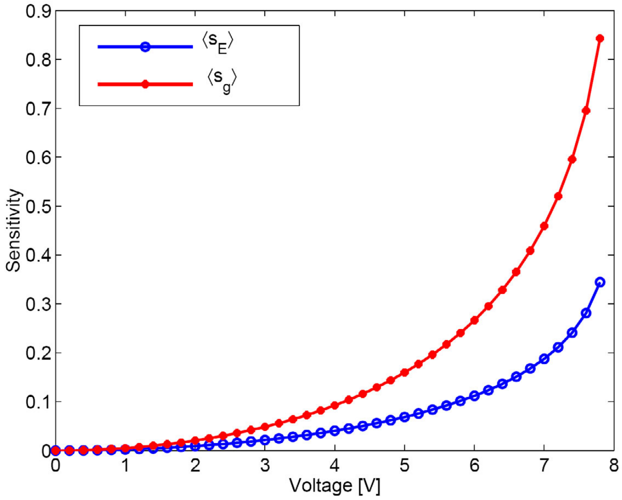

Figure 6: Sensitivity of tip deflection to the variations in Young's modulus and gap (using GPC based approach [5]).

the probability increases from 0 to 1. From a design viewpoint, we observe that the probability of pull-in for $V = 9.2\,V$ is 0.47. Using this, one can argue that if the switch was designed for the mean values of the parameters (given by the realization S0), the probability that the switch may fail to operate is 53%. It can also be said that if the operating voltage of the switch is specified as $V = 10.5\,V$, the probability that the switch will operate successfully is 88%, which may be acceptable. In the absence of such information, one may have no choice but to operate the switch at $V = 11.4\,V$, which will lead to higher power consumption. Such observations can be effectively used to design reliable and efficient MEMS devices, which would give desired performance and improve reliability.

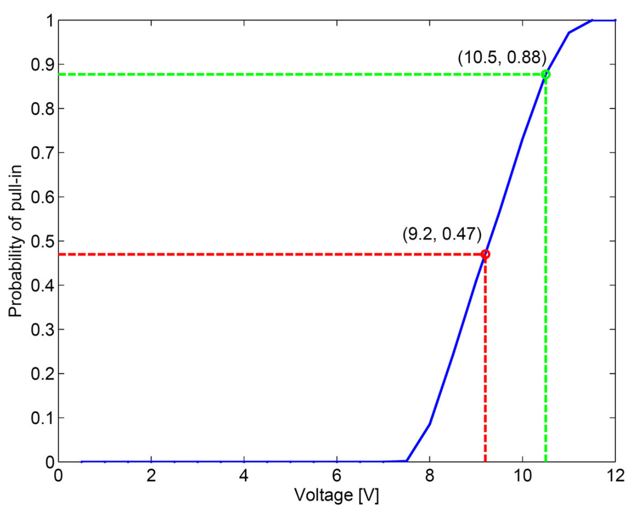

Figure 7: Probability of pull-in with various applied voltage.

In Figure 8 we plot the probability density functions (PDF) of the vertical tip displacement for various applied voltages. As expected, for voltages between 7.3 V and 11.4 V, we observe two peaks in the PDF, which indicates a non-zero probability of pull-in.

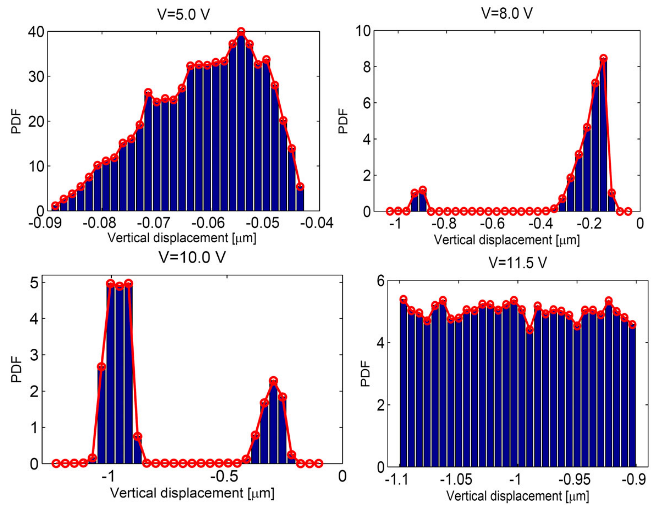

Figure 8: Probability density function (PDF) of vertical tip displacement for various applied voltage.

CONCLUSION

We presented a stochastic framework to quantify the effect of uncertain parameters on the performance of MEMS devices. The approach can also be used to identify those design parameters which are critical to the performance of the device. We demonstrated by considering the example of a MEMS switch that this framework can be used to design effective and reliable devices, which would give desired performance and improve reliability.

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